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DEVELOPMENT OF ANALYTICAL AND EXPERIMENTAL DYNAMIC MODEL REPRESENTATIONS FOR THE PREDICTION OF BASEBALL BAT PERFORMANCE

Amanda Askin
C. Kennan Crane
Kenneth Marek
Dr. Peter Avitabile
Los Alamos Dynamics Summer School
Los Alamos National Laboratory
Los Alamos, New Mexico

ABSTRACT

In recent years, sporting goods companies have made increasing use of structural analysis, particularly in the area of dynamics, to optimize the performance of baseball bats. This enhanced performance has created safety concerns related to ball velocity and has necessitated the development of regulations relating to bat performance and a means of accurately predicting this performance.

A detailed finite element model is developed along with an analytical reduced-order model that captures the essential bat dynamics. Experimental modal testing is performed to validate these models. In addition to the analytical representations, a mass, damping and stiffness model is derived from the measured experimental data and compared to the analytical models. This enables the quick development of a model to predict performance characteristics of the baseball bat on the playing field.

INTRODUCTION

A major difficulty in the testing and certification of baseball bats is that the process is very time consuming. Physical testing requires a large number of tests and generally uses large, expensive equipment and analytical models require a great deal of time to create and must then go through many test simulations. Often the desired result from the model or test is the achievable ball exit velocity, which determines the bat's suitability for use. Another problem that exists is that bulky equipment and large analytical models are not practical to bring to the field to test bats in situ to verify their performance. This suggests that a new approach is needed to verify equipment.

Nathan [1] discusses the importance of the contributions of vibrations to the exit velocity of the ball, and also notes that for hits far from the bat support, the resultant ball velocity is independent of the type of support used. In addition, Nathan, Russell, and Smith [2] investigate the contribution of vibrational hoop modes to the dynamics of the bat-ball interaction in hollow bats.

A new approach, suitable for in situ verification of bats, using a reduced order model obtained from experimentally derived data is presented. The ultimate purpose of the model is to determine the system response characteristics, specifically the exit ball velocity. This approach develops a reduced order mass and stiffness matrix of the bat configuration from inverse processing of the experimentally acquired data. This model may then be used in a forced response impact model to determine ball exit velocity.

In order to assure that reasonable physical matrix characteristics are obtained, the reduced order model is compared to the characteristics of the detailed design models developed in the laboratory. Through model reduction [3,4,5], model expansion [4], correlation [6,7] and analytical model improvement techniques [8], the proposed approach is verified to assure that reasonable physical characteristics and response characteristics are obtained.

A test case is shown for one bat configuration to identify the process required to perform this new in situ approach. The work in this paper presents the reduction, expansion, correlation and analytical model improvement; subsequent work will address the response analysis to verify bat characteristics. Discussion of the issues related to the numerical processing of this analytically and experimentally derived data is presented.

THEORY AND PROPOSED METHOD

Each of these methods is only summarized here; details of the techniques are contained in the respective references. The following sections describe the theory related to the reduction and improvement of the baseball bat models.

Model Reduction:

Model reduction is a method of transforming large degree of freedom models to model representations with fewer degrees of freedom. This process enhances the computational speed of the model and allows for a larger number of simulations in a shorter period of time. For this work, the reduction is specifically done for the development of lower order models that are determined experimentally. One concern with all reduction schemes is distortion of the reduced model dynamic characteristics.

Several methods for reducing the analytical models have been utilized. Three common methods are Guyan [3], SEREP [4], and a Hybrid method [5]. In these methods, the relationship between the full set of degrees of freedom and a reduced set of degrees of freedom can be written as:

$$\{X_n\}=[T]\{X_a\} \tag{1}$$

All three methods require the formation of a transformation matrix that can project the full mass and stiffness matrices onto a smaller space. This smaller space is defined by a reduced mass matrix and a reduced stiffness matrix. These matrices are related to their respective full space equivalents as:

$$[M_a]=[M_n]^T[T][M_n] \tag{2}$$

$$[K_a]=[K_n]^T[T][K_n] \tag{3}$$

The Guyan reduction process [3] forms the transformation matrix as:

$$[T_s]=\begin{bmatrix} [I] \\ -[K_{dd}]^{-1}[K_{da}] \end{bmatrix} \tag{4}$$

The SEREP reduction process [4] produces reduced matrices for mass and stiffness that yield the exact frequencies and mode shapes as those obtained from the eigensolution of the full size matrix. The SEREP transformation is formed as:

$$[T_U]=[U_n][U_a]^g \tag{5}$$

The Hybrid method [5] utilizes the accuracy of the SEREP method and seeds the reduced matrices with reduced Guyan matrices to insure that the resultant reduced matrices are fully ranked for all cases. The Hybrid method transformation matrix is:

$$[T_H]=[T_S]+[[T_U]-[T_S]][[U_a][U_a]^T[T_U]^T[M_n][T_U]] \tag{6}$$

Analytical Model Improvement :

These reduced physical analytical models are adjusted using experimental results through an inverse calculation.

From basic modal representation of the system, modal mass and stiffness are evaluated as:

$$[E]^T [M][E] = [\overline{M}] = [I] \tag{7}$$

$$[E]^T [K][E] = [\overline{K}] = \left\lfloor \Omega^2 \right\rfloor \tag{8}$$

Using a generalized inverse of these equations, the mass and stiffness can be estimated as:

$$\left[E^g\right]^T [I]\left[E^g\right] = [M] \tag{9}$$

$$\left[E^g\right]^T \left\lfloor \Omega^2 \right\rfloor \left[E^g\right] = [K] \tag{10}$$

This works well for cases where the number of relevant modes is equal to the number of degrees of freedom. When the number of degrees of freedom is greater than the number of modes, seeding matrices can be used in conjunction with the experimental mode shapes as described in reference 8.

Improvement to these results can be performed as:

$$[M_I] = [M_S] + [V]^T \left[I - \overline{M}_S\right][V] \tag{11}$$

$$[K_I] = [K_S] + [V]^T \left[\,`\omega^2 + \overline{K}_S\right][V] - \left[[K_S][E][V]\right] - \left[[K_S][E][V]\right]^T \tag{12}$$

where:

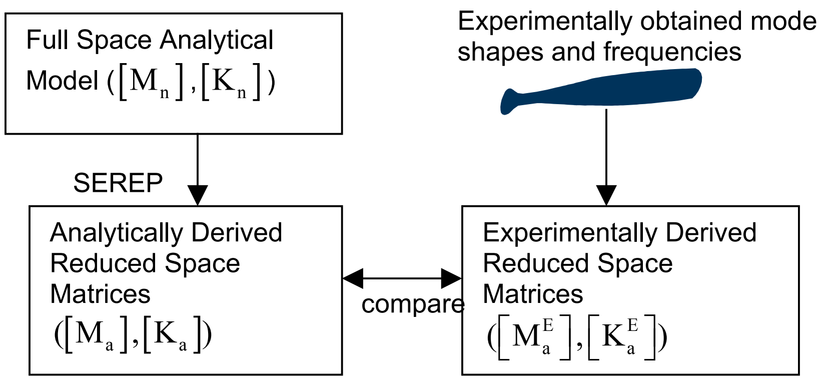
$$[V] = \left[[E]^T [M][E]\right]^{-1} [E]^T [M] = \left[\overline{M}\right]^{-1} [E]^T [M] \tag{13}$$

Proposed Method for BAT Performance Model From Experimental Data :

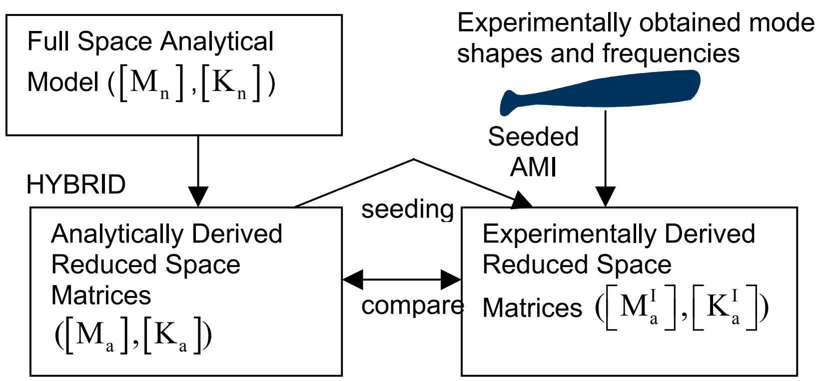
The techniques for model reduction and analytical model improvement are presented above. The methodology for development of a bat model from experimental data collected in situ on the playing field for quick bat validation is presented herein. The proposed method is shown in the Schematic A and B below.

The concept is to develop a mass and stiffness matrix by direct inversion of the experimentally measured frequencies and mode shapes as shown in the right half of Schematic A. The development of the mass and stiffness will only have physical meaning if the number of measured modes equals the number of measured degrees of freedom (ie, ADOF = AMOD). In this condition, the generalized inverse is a standard inverse and an exact solution exists. If this is not the case, then the matrices will be rank deficient and of limited value for the forced response study. In order to confirm the experimentally derived matrices are physically realizable, the full space model is SEREP reduced to the same size model as the experimental model. The comparison of the reduced analytical model and experimentally derived model at ADOF is done to assure that similar mass and stiffness characteristics are obtained from the proposed method.

In the case where the number of measured modes is less than the physical number of degrees of freedom (ie, ADOF > AMOD), then the rank of the matrices is of concern. In order to preserve the full rank of the reduced matrices, the Hybrid reduction scheme is utilized for the development of the reduced model. This fully ranked reduced model is used as a seeding matrix for the analytical model improvement procedure. In cases where ADOF > AMOD, the Hybrid technique is the preferred method for development of the reduced matrices; the SEREP process does not produce fully ranked matrices for this condition. Schematic B shows the concept of the mass and stiffness development from the direct inverse of the experimentally measured frequencies and mode shapes with a fully ranked seeding matrix.



Schematic A: Flow chart of reduction and validation process for ADOF equal to AMOD.



Schematic B: Flow chart of reduction and validation process for ADOF greater than AMOD.

For the initial work investigated under this study, only the approach described for Schematic A was performed and only for a limited number of cases. Much more work still needs to be explored for this newly proposed approach but is well beyond the scope of effort expended on this study.

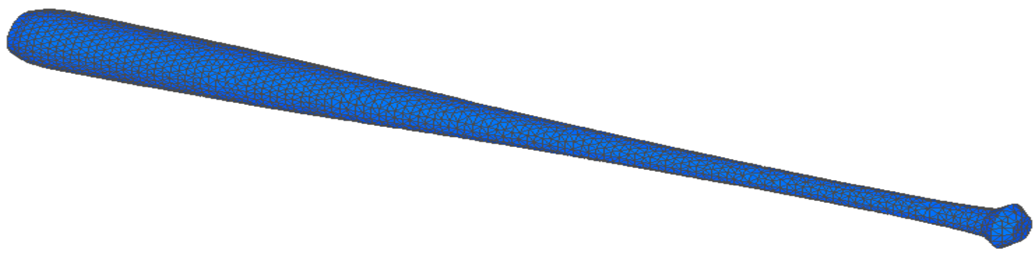
The following sections discuss some of the models developed, tests performed, reduced models investigated and experimentally models obtained from the proposed approach.

WOOD BAT MODELS DEVELOPED

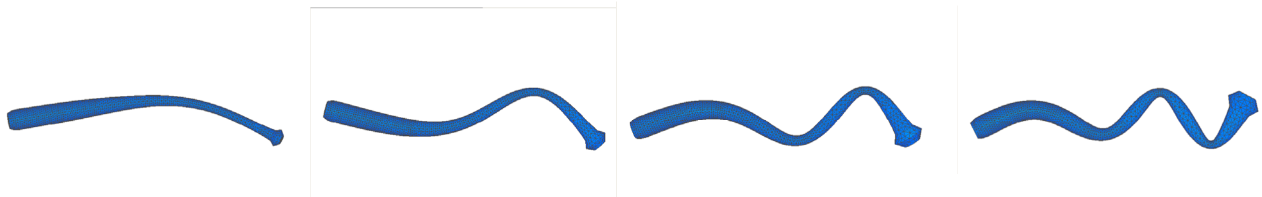
A finite element model, an analytical beam model, and an experimental model were created in order to validate the applied model reduction procedures. A 33 in (83.82 cm) wood bat was used in both the computer and experimental models. The bat was found to have a mass of $4.844*10^{-3}$ slinch (848.4 g). For the computer models, a published value of $1.74*10^{6}$ psi (12,000 MPa) was used for the elastic modulus [9], and the density was calculated to be $6.56*10^{-5}$ slinch/in^3 (701 kg/m^3). These computer models were compared with each other and with the experimental data to ensure accuracy of the models. In all models, free-free boundary conditions were simulated based on the work of Nathan [1], which indicates that for impacts on the bat far from the bat supports (the batter's hands), the method of support has no effect on the resultant ball velocity.

Wood Bat Finite Element Model

A finite element analysis was performed for the wood bat using Abaqus CAE [12]. The wood was assumed to be isotropic. The geometry was created by rotating the profile of the bat around its central axis. The model was then meshed to create over 15,000 nodes and over 70,000 solid linear tetrahedral elements. An eigenvalue solution was performed to predict the first several bending modes of the bat. Torsional modes were neglected in this study because they are not of interest in the ball-bat interaction. The bending modes predicted were 149, 495, 1010, 1650, 2420, 3320, and 4320 Hz. The bat geometry and the first four bending modes are illustrated in Figure 1.



(a) Undeformed Bat



(b) First Mode Shape (c) Second Mode Shape (d) Third Mode Shape (e) Fourth Mode Shape

Figure 1: Finite element model of a wood bat

Wood Bat Beam Model

A beam model of the wood bat was created in MATLAB [13]. The geometric properties were identified by a 6th-degree polynomial that was fit to measurements made at half-inch (1.27 cm) increments along the length of the bat. In order to construct full size mass and stiffness matrices, the bat was modeled as a beam of varying cross section with the section properties computed from the dimensional information. The boundary conditions were approximated by the placement of relatively weak support springs at one-half-inch (1.27 cm) toward the center on each end of the bat. This model had 67 nodes, each with one translational and one rotational degree of freedom for a total of 134 degrees of freedom. Figure 2 shows the resulting mode shapes and frequencies for the first seven modes.

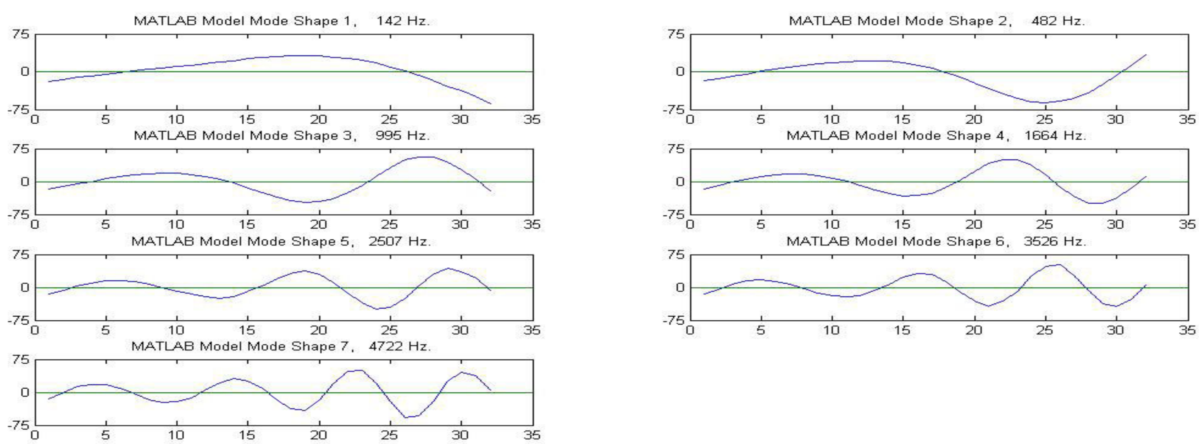


Figure 2: Analytical mode shapes and frequencies from MATLAB.

Wood Bat Experimental Model

The wood bat was suspended for experimental modal testing to simulate a free-free boundary condition as shown in Figure 3. In order to capture the anisotropic properties of the bat, two stationary accelerometers were placed at one inch (2.54 cm) from the barrel end of the bat one with the grain and one against the grain as seen in Figure 4.

Figure 3: The bats tested were suspended in order to simulate a free-free boundary condition.

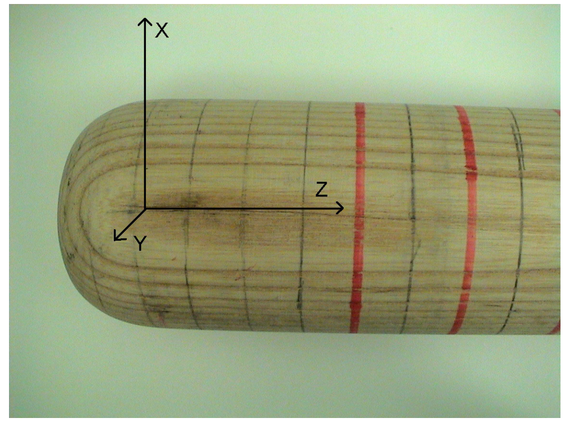


Figure 4: The coordinate axes selected with the z-axis along the length of the bat.

The bat was tested using an impact technique. Measurements were made at one-inch (2.54 cm) intervals along the length of the bat in both the X and Y directions. The data was collected and the frequency response function (FRF) was calculated with the Dactron Photon [10] data acquisition system using a frequency range of 2 kHz. No damping window was required because the time record was selected to observe the entire transient. The data was then exported to ME'ScopeVES [11] to calculate the resonant frequencies and associated mode shapes.

Due to the anisotropic properties of the wood, distinct separate modes in the x and y direction were observed. Table 1 lists the first seven frequencies observed in each direction.

Table 1: Anisotropic Wood Bat Frequencies (Hz.)

Mode	X-direction Impact	Y-direction Impact
1	157.7	159.7
2	517.7	525.6
3	1030	1043
4	1626	1659
5	2303	2361
6	3019	3113
7	3787	3905

Since these frequencies were consistently within 5% of each other for any given mode and in order to simplify later model comparisons, only the X-direction data is considered in the rest of the experiment. The X-direction characteristics are also more reasonable for looking at field performance because batters typically hit the ball with the grain of the bat. Figure 5 shows the first seven mode shapes and their associated frequencies for X-direction bat excitation.

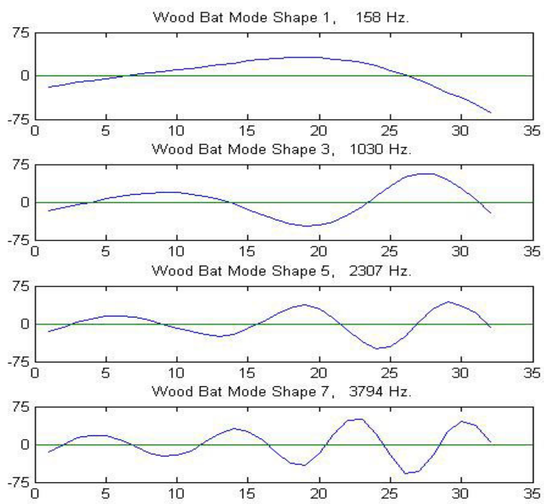


Figure 5: Experimental frequencies and mode shapes.

Model Comparison

In order to validate the beam and finite element models, the mode shapes and frequencies of the models were compared to each other, and then each was compared to the test data. These computer models also created mass and stiffness matrices that could be later compared to the mass and stiffness matrices calculated with the Analytical Model Improvement (AMI) method from the experimental mode shapes. Table 2 compares the calculated frequencies from the three models for the first seven bending modes.

Table 2: Comparison of Frequencies for First Seven Modes from Various Models (Hz.)

Mode	Experimental Model	Finite Element Model	Beam Model
1	157.65	148.88	142.37
2	517.68	495.12	482.31
3	1030.1	1006.8	994.81
4	1627.4	1648.4	1664.3
5	2306.6	2424.9	2507.0
6	3027.6	3321.0	3525.9
7	3794.2	4321.7	4722.4

In order to determine the correlation between models, both the Modal Assurance Criterion (MAC) and the Pseudo Orthogonality Check (POC) were used to compare each pair of models.

Beam Model Compared to Finite Element Model

Figure 6 shows the MAC of the mode shapes calculated from the beam and finite element models. The lowest of these values is 0.9726 which indicates a very high confidence level in the match of the mode shapes. The POC in Figure 7 has values above 0.90 for all of the diagonal values except the seventh, which is 0.8936. These high values indicate that these mode shapes not only correlate well but also have very similar shape values. These results are expected given that the models were created from identical geometries and physical bat characteristics, but they serve as a good check of the correlation methodology before it is implemented on experimental data.

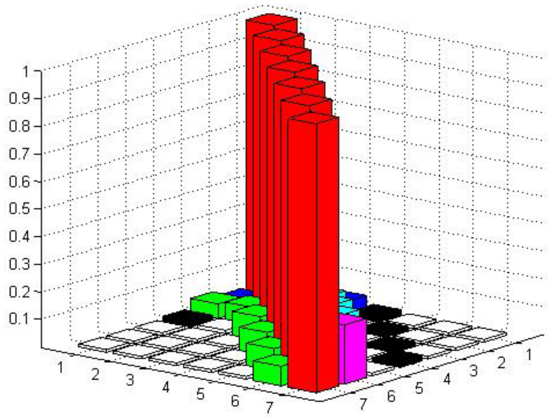


Figure 6: MAC of beam and finite element mode shapes.

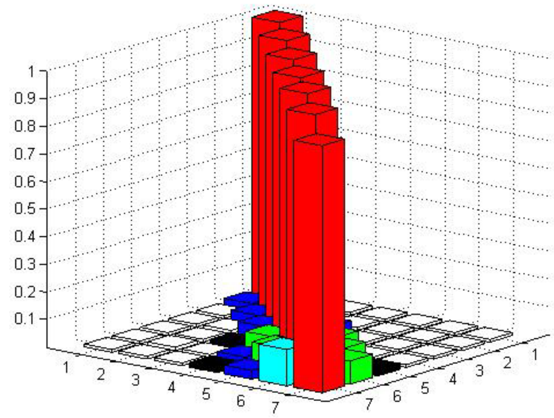


Figure 7: POC of beam and finite element mode shapes.

Experimental Model Compared to Finite Element Model

The MAC of the experimental model compared to the finite element model shown in Figure 8 shows a very good correlation between the experimental mode shapes and those derived from the ABAQUS model. The first five MAC values are above 0.99 and even the lowest is 0.9650. The POC values shown in Figure 9 show very good correlation but the values being greater than 1.0 suggest that there is a scale factor that is expected to be associated with the calibration of the experimentally derived data. The overlay of the mode shapes shown in Figure 10 supports the idea that a scale factor is responsible for this inconsistency.

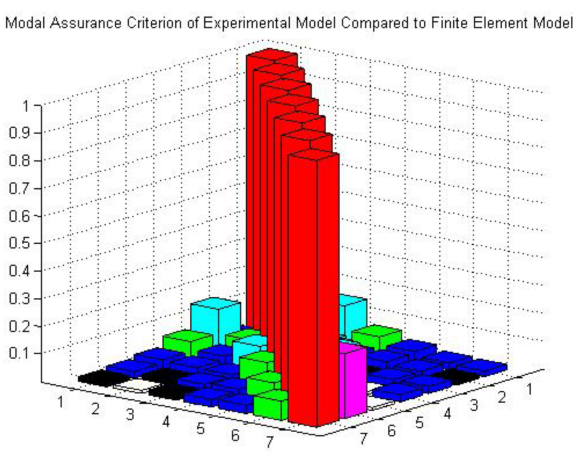


Figure 8: MAC of experimental and finite element mode shapes.

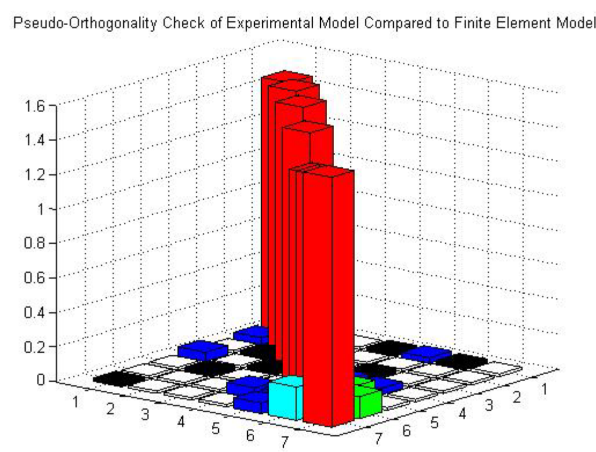


Figure 9: POC of experimental and finite element mode shapes.

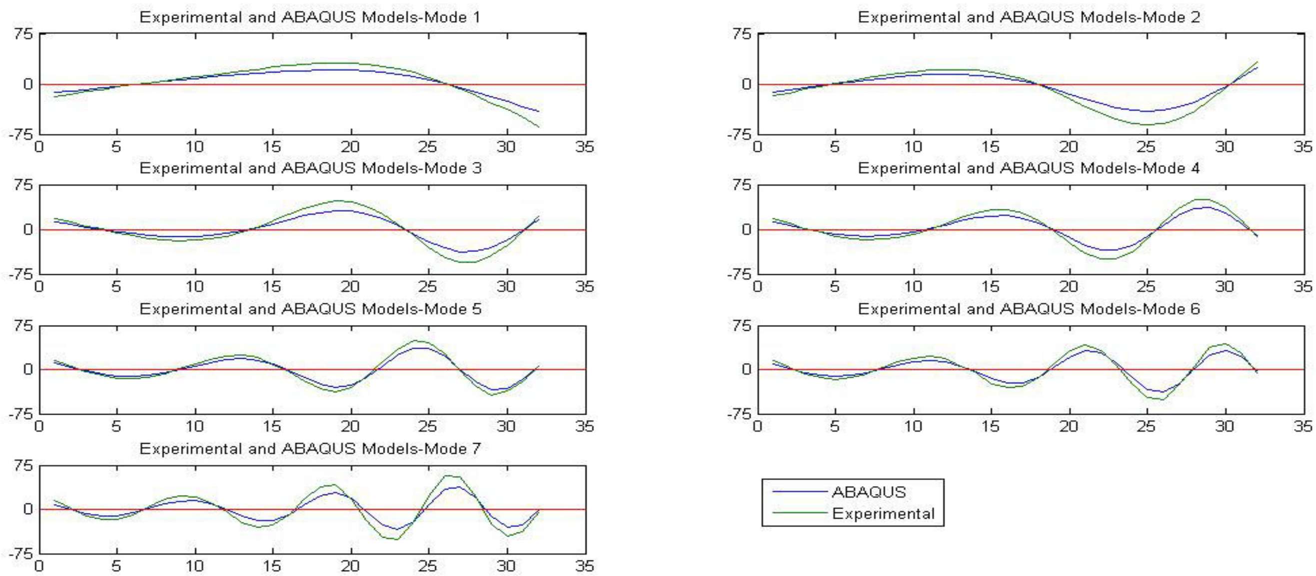


Figure 10: Overlaid mode shapes from experimental and finite element data.

Experimental Model Compared to Analytical Model

Figure 11 shows the MAC of the experimental model when compared to the analytical beam model. These values are again high with the first six modes having values above 0.93. The MAC shows that the correlation is not as strong between the experiment and the beam model as it is between the experiment and the finite element model. This is expected due to the more detailed nature of the finite element model. The POC between these two models shown in Figure 12 again shows the scale error in the data but the correlation is very good. Figure 13 again shows the scalar nature of the discrepancy by overlaying the mode shapes from the beam model and experimental data. Due to time constraints, this error could not be corrected during the course of this work; further study is needed to identify the source of this problem.

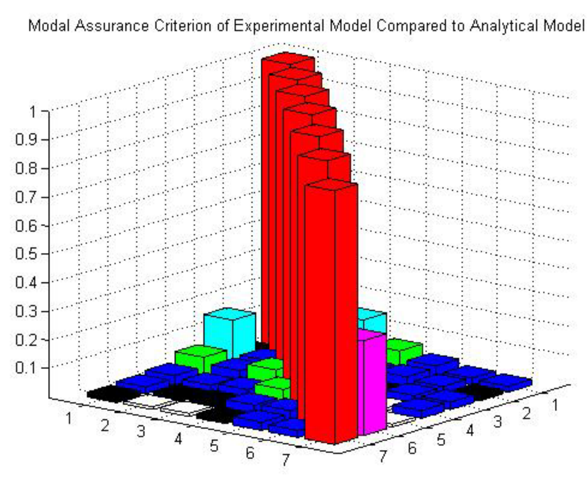


Figure 11: MAC of experimental and beam mode shapes.

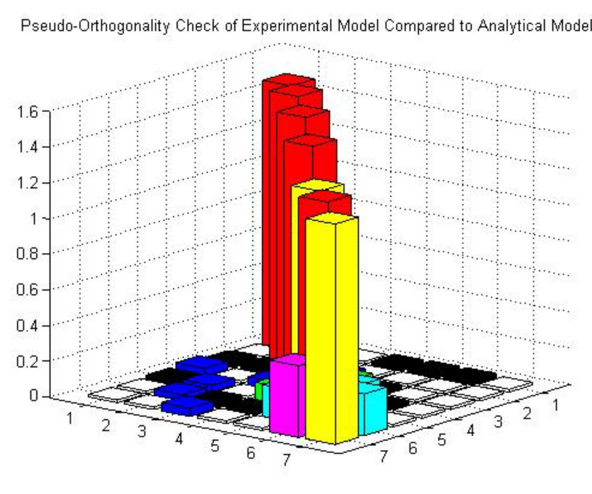


Figure 12: POC of experimental and beam mode shapes.

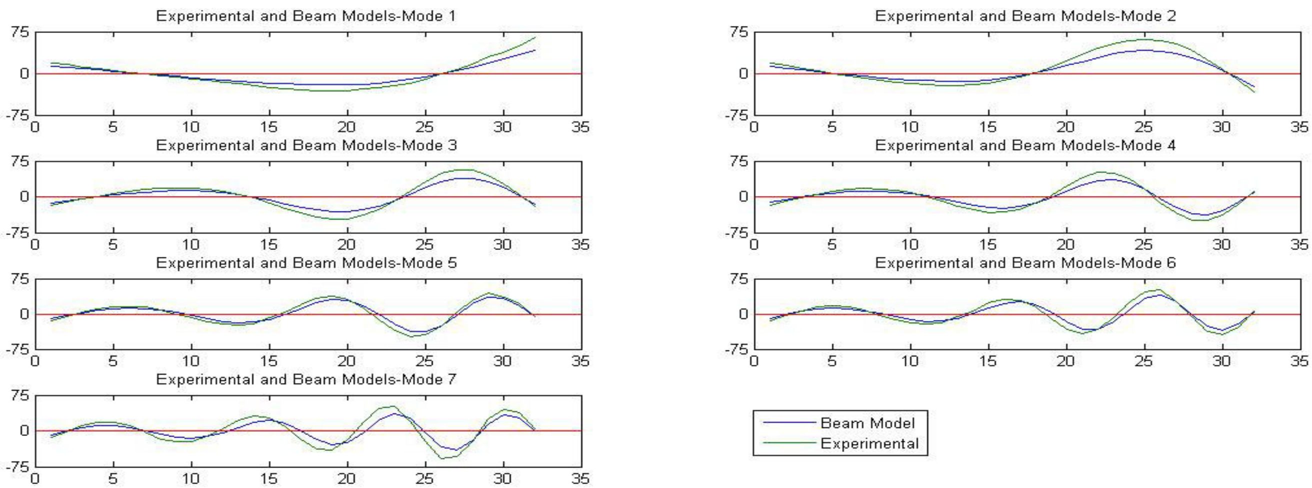


Figure 13: Overlaid mode shapes from experimental and beam model data.

EXPERIMENTALLY DERIVED MODELS DEVELOPED

The concept of the proposed approach is to develop an experimentally derived mass and stiffness from the measured frequencies and mode shapes. To illustrate the approach, a simulated model was developed. Due to time constraints, the actual measured data was not entirely completed and the actual experimental validation of the proposed approach could not be performed.

Simulated Experimental Model

In order to simulate the expected variations for a typical bat and to allow for investigation of the proposed approach, the original analytical model was slightly altered to reflect an increased mass at the knob end to simulate an experimental bat that varies slightly from the original analytical bat model. This simulated experimental model originated from the same properties as the generic typical bat model. With this starting set of mass and stiffness matrices, the Hybrid method was utilized to reduce the matrices to seven degrees of freedom of interest. At the reduced matrix scale, the mass of the end knob on the bat was increased in an effort to simulate a possible alteration from the typical bat. This might reflect an actual “on field” condition where a “juiced bat” is suspected in an actual game.

The reduced mass and stiffness matrices were used to find the reduced set of frequencies and mode shapes. Through the AMI method, these frequencies and mode shapes were used to calculate reduced mass and stiffness matrices. The original reduced mass and stiffness from the first analytical model were used to seed the resulting improved matrices. This process forces the resultant small space mass and stiffness matrix approximations to be fully ranked while still being representative of the simulated experimental model. The HYBRID reduced matrices from the initial typical bat model were then compared with the matrices that resulted from the AMI process. These small space mass and stiffness matrices were compared in an effort to discern the effect of a mass perturbation on the mass and stiffness matrices that could result from a typical field test.

Figure 16 shows a reduced mass matrix for the analytical model. Figure 17 shows the perturbed reduced model used for the simulation; a mass was added at the knob of the bat to simulate some modification of the bat that might be added to enhance the performance of the bat (“juiced bat”). The experimental mode shapes were obtained from an eigensolution with the perturbed reduced mass. The analytical model improvement process was performed and the mass discrepancy is shown in Figure 18. The mass that was added to the knob of the bat is very clearly identified. Since there was no change in the stiffness in the simulation, there was no measurable change in the stiffness following the analytical improvement process (these matrices are not shown since there is no change observed).

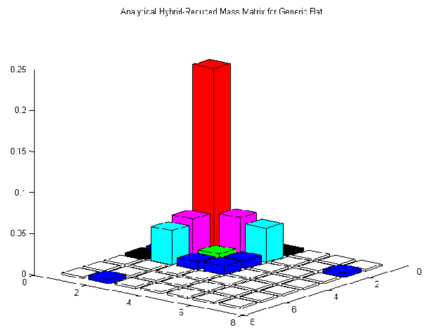


Figure 16: Analytical Reduced Mass

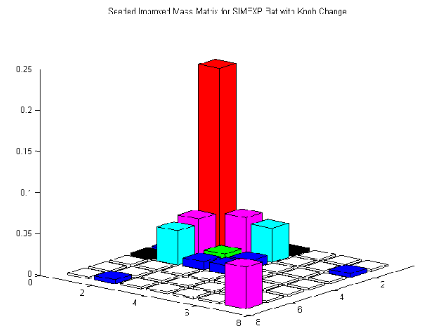


Figure 17: Perturbed Analytical Reduced Mass

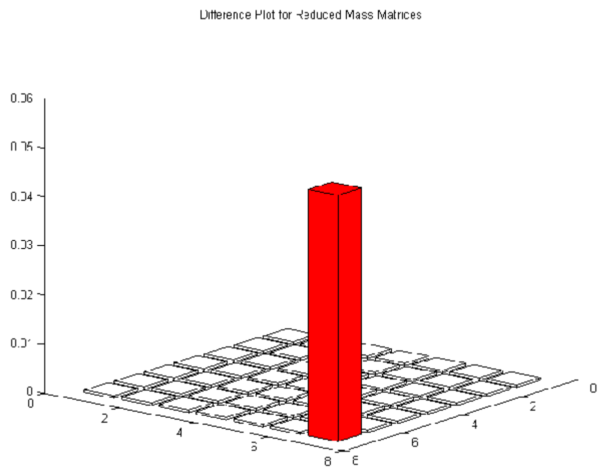


Figure 18: Mass Discrepancy Resulting from Analytical Model Improvement

Clearly, the proposed approach shows potential for identification of changes to bat characteristics that can be very quickly measured in situ on the playing field. There are many issues to address related to sensitivity of the proposed approach to a variety of different factors and many studies are yet to be performed.

FUTURE WORK

The development of the mass and stiffness matrices from experimentally derived frequencies and mode shapes has proved that the proposed technique is feasible. Due to time constraints, many cases were not possible to be developed and performed over the limited time available in the abbreviated six week session at Los Alamos Labs during the Summer Session in 2004.

Effort needs to be expended in several areas to prove the adequacy and robustness of the proposed technique. Studies of the reduced models when the number of modes is not equal to the number of measured degrees of freedom need to be performed to better understand the effects of the selection of degrees of freedom for the reduced model. The further use of actual experimental data needs to be addressed and the discrepancies that result from scale issues observed needs to be clarified and fully understood. The desired characteristic for evaluation is actually the response (exit velocity) of the bat; this needs to be evaluated to develop a metric so that bats can be evaluated on the playing field. Finally, the wood bat was evaluated but the extension to the aluminum bat needs to be studied.

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CONCLUSIONS

Evaluation of bat performance characteristics needs to be performed in situ on the playing field. A new approach to determination of bat performance has been proposed and shows promise as an alternative mechanism for bat performance verification on the playing field.

The new approach utilizes a combination of model reduction, model expansion, and analytical model improvement at the reduced order model space to develop an experimentally derived mass and stiffness matrix from measured frequencies and mode shapes. The proposed approach needs further evaluation but shows promise as a viable approach to solve the problem of "juiced bats" on the playing field.

NOTATION

Matrix

$[\mathrm{M}]$ analytical mass matrix
$[\mathrm{K}]$ analytical stiffness matrix
$[\mathrm{U}]$ analytical modal matrix
$[\overline{\mathrm{M}}]$ diagonal modal mass matrix
$[\overline{\mathrm{K}}]$ diagonal modal stiffness matrix
$[\mathrm{T}]$ transformation matrix
$[\mathrm{E}]$ experimental modal vectors

Vector

$\{\mathrm{X}\}$ displacement

Subscript

n full set of finite element dof
a tested set of experimental dof
d deleted (omitted) set of dof
s seed matrix
I improved or updated matrix

Superscript

T transpose
g generalized inverse
-1 standard inverse

$$[V]=\left[[E]^T[M][E]\right]^{-1}[E]^T[M]=\left[\overline{M}\right]^{-1}[E]^T[M]$$ generalized inverse

Acronyms

ADOF Reduced degrees of freedom
AMI Analytical Model Improvement method
AMOD Modes used for reduction process
MAC Modal Assurance Criteria
POC Pseudo Orthogonality Check
SEREP System Equivalent Reduction Expansion Process

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